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Comparison of Different Techniques for Voltage Stability Analysis of Power Systems

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Abstract— **It is always crucial for power systems to be Voltage stable to ensure reliable and secure power to its customers. However, voltage stability of modern power systems is being challenged more and more as the structure of power systems is being changed. Voltage stability analysis techniques were introduced to understand about voltage stability problem and to take necessary precautions to prevent voltage collapse. There have been different techniques proposed in the literature and this paper compares some of these techniques. Finally, IEEE 9 bus systems is used to evaluate results for voltage stability using 6 different techniques described in this paper. A ranking of buses in IEEE 9 bus system is given using evaluation from each technique. The impact of action of under load tap changing transformer (ULTC) for voltage stability is also observed for the test system.**

I. Introduction

Voltage stability is defined as "the ability of the power system to maintain steady voltages at all buses of the system after being subjected to a disturbance from a given initial operating point" [1]. Voltage stability can be further divided into two areas as large disturbance voltage stability and small disturbance voltage stability.

Large disturbance voltage stability involves system's ability to maintain voltages following a large disturbance such as tripping of a line or a power plant. Non-linear characteristics of the power system components are taken into consideration in this analysis. Small disturbance voltage stability involves system's ability to maintain voltages for small changes such as incremental change in load. It can be studied by both linear approximations and non-linear characteristics of the power system. A voltage stability issue may remain from several seconds to few minutes. Therefore, both large disturbance and small disturbance voltage stability are further divided into long term and short-term stability [1].

Main reason for voltage instability is inability of the power system to provide required reactive power demand. Reactive power (Q) versus bus voltage (V) is a direct measure of voltage stability of the system. If an electric power system is voltage stable, it should show positive Q-V response for all buses in the system. If there is at least one bus which shows negative QV response, then the whole system is considered as voltage unstable.

A voltage collapse is a situation that leads to unacceptable voltages in a part of the power system [1]. There is always a triggering event that leads a power system towards voltage collapse. Most common triggering events are tripping of a transmission line, tripping of a generator or a system overload. Out of the 7 blackouts discussed in [2], triggering events for 3 blackouts were a short circuit causing trip of a transmission line, 3 other blackouts were triggered by tripping of a power plant and remaining one was triggered by system overload.

There are several factors to be considered when carrying out a voltage stability study. Following factors have the strongest influence on the voltage stability of a system [3]. Transmission network and power transfer, Generator VAR control limits, Load characteristics, Reactive power compensation devices, Action of ULTC transformers.

In understanding the mechanism of voltage stability, two main approaches can be identified as static analysis and dynamic analysis [3]. Static analysis uses the snapshots of the power system in different times to calculate voltages. As the name itself suggests, static analysis considers only static aspects of the power system. Hence time variant differential equations are set to zero. Static analysis is mainly carried out via load flow analysis techniques. These are two main techniques of static analysis as PV, QV analysis and VQ sensitivity analysis.

Dynamic analysis is very much similar to transient stability analysis of the power system. In this method, the system is represented using a set of differential equations and algebraic equations. Solution of the transient stability gives the response of the system for different load levels and contingencies.

Although both static analysis and dynamic analysis can provide a good insight to the voltage stability problem, it does not specifically indicate how far the system is from voltage collapse. This led researchers to find voltage stability indices

(VSI) or proximity to voltage stability. There are mainly two approaches for VSIs proposed in the literature as follows [4] as Jacobian matrix and system variable based VSIs and Bus, line and overall system based VSIs.

Singular value decomposition technique, eigenvalue decomposition technique and test functions are explained in [5] as Jacobian matrix based VSIs.

Jacobian based VSIs became time consuming for large power systems because it requires recalculation of Jacobian. Therefore, simple line or bus based VSIs were introduced. In [4] discusses 29 different indicators based on bus, line and overall equations of the power system.

II. VOLTAGE STABILITY ANALYSIS

This section briefly discusses static and dynamic voltage stability analysis techniques and VSIs (Jacobian matrix based and line based VSIs).

A. *Voltage stability analysis techniques*

1) *Static analysis*

Overall system can be represented by a set of first order differential equations and a set of algebraic equations as follows.

$$\dot{x} = f(x, V) \quad (1)$$

$$I(x, V) = Y_N V \quad (2)$$

In equations (1) and (2), x is the state Vector, V is the bus voltage vector and Y_N is the bus admittance matrix.

In static analysis differential vector $\dot{x}$ is set to zero, meaning that the system is in steady state, then the system is reduced to purely algebraic equations.

This is simply done by load flow techniques. PV, QV analysis and VQ sensitivity analysis comes under static analysis.

a) *PV and QV analysis*

For PV and QV analysis, a bus bar is selected and the load (P or Q values) of that bus is increased separately to obtain a set of load flow results. All other operational parameters of the power system are kept constant while only the selected bus is stressed along either P or Q values. At the point of voltage collapse, load flow algorithm fails to converge.

PV and QV analysis does not provide good insight into the overall stability of the power system and they are time consuming as well.

Another approach of static analysis is VQ sensitivity analysis.

b) *VQ sensistivity analysis*

As explained in [3], equation for reduced Jacobian matrix can be written as follows.

$$J_R = \left(J_Q \ - J_Q \ \times J_P^{-1} \times J_P \ \right) \quad (3)$$

$$\Delta V = J_R^{-1} \Delta Q \quad (4)$$

The value of J_R^{-1} indicates the voltage stability of the system. A positive value of J_R^{-1} represents a stable system. (Voltage increases as reactive power is injected). If value of J_R^{-1} is high, then sensitivity is high, which means system is less stable. Lesser values of J_R^{-1} (closer to zero) gives a more stable system.

2) *Dynamic analysis*

Due to the non-linear feature of power system components, it is hard to use classical methods to solve differential equations in a transient response. Therefore, numerical integration techniques must be used. To do a dynamic study, a disturbance in the system should be simulated. After the disturbance, voltages at different bus bars in the power system begins to fluctuate. Voltage response against time is the output for dynamic study related to voltage stability.

B. *VSIs*

1) *Jacobian matrix and system variable based VSIs*

a) *Singular value decomposition (SVD) technique [6]*

This technique uses SVD for Jacobian matrix. Singular values of J can be written as,

$$J = V \ \ U^T = \sum_{i=1}^{2(n-1)} u_i s_i v_i^T \quad (5)$$

Some of the features of above equations are as follows.

- U is the left singular vector, V is the right singular vector and S is the singular vector
- n is the number of system buses. u_i, s_i and v_i^T are i[th] columns of matrices V,S, U^T.
- U and V are unitary matrices. ($UU^T = U^T U = \mathbb{I}$, $VV^T = V^T V = \mathbb{I}$)

Above equation can be rearranged in the following way,

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \sum_{i=1}^{2(n-1)} s_i^{-1} v_i u_i^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (7)$$

Near voltage collapse point, only last singular (*s*) value and its right and left singular vectors are of interest because it is the least one [6]. Therefore only last *s*,*v*,*u* values are considered (values corresponding to last bus).

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \delta_{2(n-1)}^{-1} v_{2(n-1)} u_{2(n-1)}^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (8)$$

At this point,

$u_{2(n-1)}$ provide details about the power mismatches $\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$.

$v_{2(n-1)}$ provide details about critical buses.

By analysing these two vectors, weak buses and most sensitive directions for power injections can be identified. Since singular values are arranged in decreasing magnitude, least singular value ($\delta_{2(n-1)}^{-1}$) is always the last value, and corresponding left and right singular vectors are in the last columns of each vector. Ranking of buses in $v_{2(n-1)}$ vector (minimum to maximum) helps to identify weakest buses. Minimum value of $v_{2(n-1)}$ gives the weakest bus.

b) *Eigenvalue decomposition technique*

This approach is based on eigenvalues and eigenvectors of the matrix J_R [3].

$$J_R = \epsilon \Lambda \eta \qquad (9)$$

η – left eigenvector of matrix J_R
ϵ – right eigenvector of matrix J_R
Λ - diagonal eigenvalue matrix of J_R

$$J_R^{-1} = \epsilon \Lambda^{-1} \eta \qquad (10)$$

$$\Delta V = \epsilon \Lambda^{-1} \eta \Delta Q \qquad (11)$$

$$\Delta V = \sum_i \frac{\epsilon_i \eta_i}{\lambda_i} \Delta Q \qquad (12)$$

Each value of Λ, ϵ and η corresponds to the i^th^ mode of QV response.
Since $\epsilon^{-1} = \eta$,by multiplying both side we get,

$$\eta \Delta V = \Lambda^{-1} \eta \Delta Q \qquad (13)$$

$$v = \lambda^{-1} q \qquad (14)$$

where, $v = \eta \Delta V$ and $q = \eta \Delta Q$
For the i^{th} mode,

$$v_i = \frac{1}{\lambda_i} q_i \qquad (15)$$

Each of these eigenvalues associates with a mode of voltage and reactive power variation.
For positive λ_i, i^{th} voltage and i^{th} modal reactive power shows direct proportionality and hence the system is stable. For negative λ_i , i^{th} voltage i^{th} modal reactive power shows inverse proportionality and hence the system is unstable. At $\lambda_i = 0$, voltage collapse occurs. Higher values of λ_i gives a stable system compared to lower values of λ_i because V-Q sensitivity is less. By using eigenvectors associated with each eigenvalue, information about bus, branch and generator participation factors can be obtained.
But this technique cannot identify individual voltage collapse modes. It's a representation of VQ sensitivity in the whole system. If at least one eigenvalue is negative in the system, then the whole system is voltage unstable. However, it provides a relative measure of proximity to voltage instability.
Near voltage collapse point, eigenvalue closest to zero refers to the weakest mode of VQ response. Left and right eigenvectors associated with this eigenvalue gives information about sensitivity of power injections and critical buses respectively. Maximum value of left eigenvector corresponds to most sensitive power injections and maximum value of right eigenvector corresponds to most critical bus.

c) Line based VSIs

Theoretical basis of all line based VSIs are same. All of them are based on two bus representation of a power system. Obtaining a two bus equivalent of any multi bus power system is explained in [7]. In [4] out of 29 VSIs summarised, 18 of them are related to line based VSIs. These VSIs are either based on maximum power transfer through a line, existence of a solution for receiving end voltage equations or maximum power transfer theorem. Different assumptions are used in each indicator. About 16 of the 18 VSIs take the assumption of neglecting shunt admittance.

III. VOLTAGE STABILITY ANALYSIS ON IEEE 9 BUS SYSTEM

Following diagram shows an overview of IEEE 9 bus system. In this test system, buses 1-3 are generator buses and buses 4-9 are load buses.

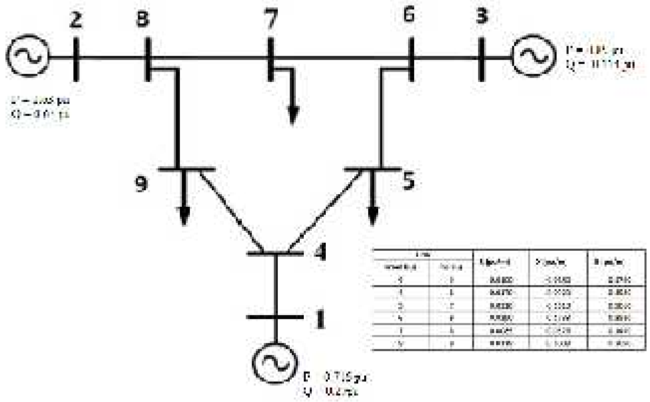

Figure 1: IEEE 9 bus system overview

1) PV analysis

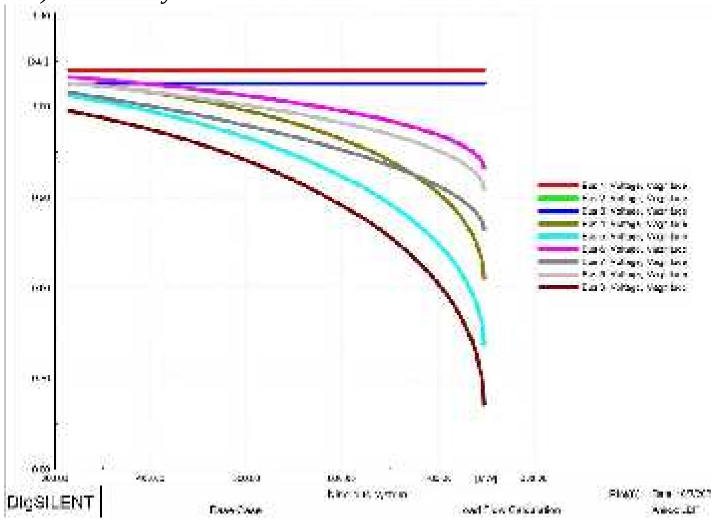

Figure 2: PV analysis of IEEE 9 bus system

In the above diagram buses 1, 2 and 3 shows flat curves. This is because they are generator buses. In buses 9 and 5 when active power is increased voltage begins to reduce drastically. Bus 6,8 shows a very robust response to the PV analysis. Bus 4 initially shows a strong response but then its voltage is decreased at a higher gradient than others at the end.

2) QV analysis

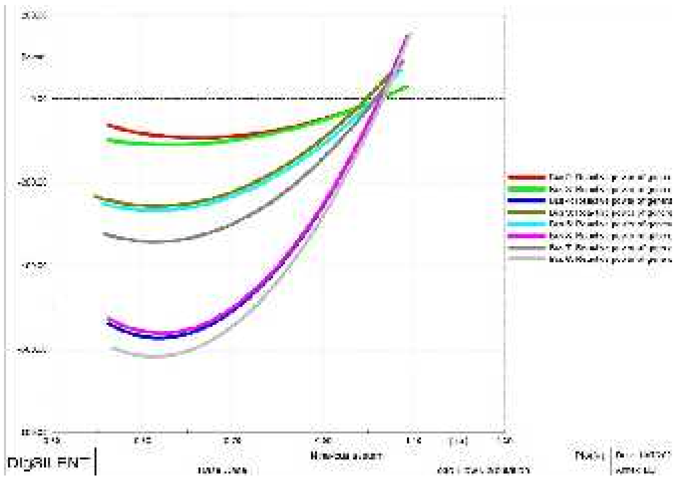
Figure 3: QV analysis of IEEE 9 bus system

Buses 4, 6, 8 show a very high gradient in QV analysis. Voltage collapse point of these three buses are at a higher value of reactive power. Meanwhile buses 5,9 show very less gradient. Although buses 1,2,3 shows a low gradient, they are not interested in the study since they are generator buses.

3) *QV sensitivity analysis*

Values of J_4 were obtained by a MATLAB program developed based on Newton Rhapson algorithm. Results for diagonal elements of J_4 are as follows. (J_4 includes only values of load buses)

Table 1: Diagonal elements of J4

Bus no	***Diagonal element***
4	40.0101
5	15.5203
6	32.4608
7	22.7718
8	35.5478
9	16.5079

A comparison is made between values of diagonal elements of J_4 and J_R as shown in the following table.

Table 2: Comparison of diagonal elements of J4 and JR

Bus	***J4***	***J_R***
4	40.0101	40.4823
5	15.5203	16.1034
6	32.4608	32.7774
7	22.7718	23.0465
8	35.5478	35.6464
9	16.5079	16.761

It is observed that in a particular bus, two values of J_4 and J_R are almost identical.

4) *Eigenvalue decomposition*

An eigenvalue decomposition was carried out for J_R^{-1} by computing eigenvalues and eigenvectors.
In MATLAB output of eigenvalues were stored in a matrix called [x y z].
Where,
x- right eigenvector of J_R^{-1}
y – eigenvalues of J_R^{-1}
z – left eigenvector of J_R^{-1}
Diagonal elements of y matrix were calculated and shown below.

Table 3: Eigenvalues of matrix Jr

Mode QV	***Eigenvalue***
1	0.1711
2	0.0789
3	0.0682
4	0.0199
5	0.0218
6	0.0282

In the above eigenvalue decomposition results, 4th eigenvalue (λ_4) is the smallest eigenvalue. Left and right eigenvectors related to this can be analysed to find the weakest buses. Table 4 shows the values for X and Z corresponding to λ_4.

Table 4: Left and right eigenvectors corresponding to lowest eigenvalue

X	***Z***
-0.6823	-0.666
0.2658	0.2614
-0.2706	-0.2767
0.3192	0.3294
-0.4339	-0.4487
0.317	0.3187

5) *Singular value decomposition*

From MATLAB, outputs of singular value decomposition are stored in a matrix called [V,S,U]. The obtained values which were obtained for diagonal of S are shown in Table 5.

Table 5: Singular values for matrix J4

Mode	***Singular value***
1	0.1712
2	0.0789
3	0.0682
4	0.0282
5	0.0218
6	0.0199

V and U matrices corresponding to last singular value is analysed to find strength of the buses and sensitivity of power injections. Values of V and U corresponding to last singular value are given in Table 6.

Table 6: V and U vectors corresponding to least singular value

V	***U***
0.6734	0.6752
-0.2637	-0.2636
0.2738	0.2735
-0.3242	-0.3242
0.4416	0.4410
-0.3192	-0.3167

6) *Dynamic Analysis*

A three phase to ground fault is simulated in each load bus bar of the system separately to identify the magnitude of the voltage dip. This was carried out using PSCAD software. From results which were obtained from Dynamic analysis, strength of bus bars were ranked from strongest to weakest.

7) *Action of ULTC transformer*

PSCAD software was used to do a dynamic analysis of IEEE 9 bus system including a tap changing transformer. The dynamic analysis was carried out for both stable and unstable conditions of the system.

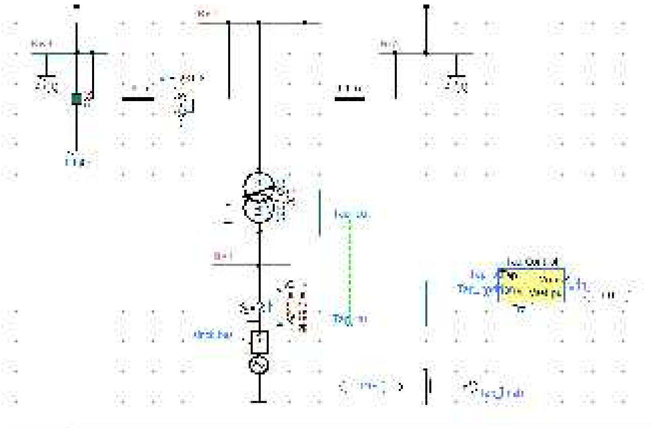

Figure 4: Dynamic study on IEEE 9 bus system including a ULTC (only a section is shown)

a) Stable condition

An extra load (in addition to existing load) of 100 MW and 30 MVAR are added to bus 9 at 5s until 70s. Effect of under load tap changing transformer action was observed. Obtained results are shown below,

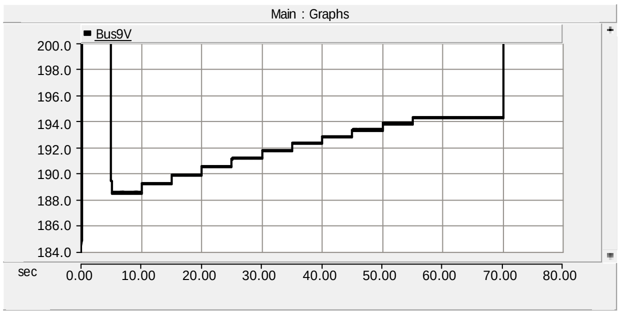

Figure 5: Voltage at bus 9 (expanded)

As seen in the above graph, as soon as the extra load was added to bus 9(at 5s), voltage of the bus was sunk to 192 Kv. Then the action of under load tap changing transformer increased the voltage in 10 steps (by reducing turns ratio) until the voltage at bus 9 reached 198 kV.

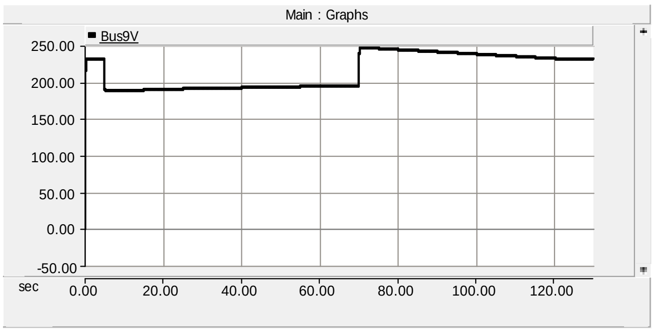

Figure 6: Voltage at bus 9

When the extra load was removed from the system (at 70s), voltage at bus 9 went above pre disturbance voltage. Under load tap changing transformer senses this over voltage and increases the turns ratio (by increasing tap position) to bring down the voltage.

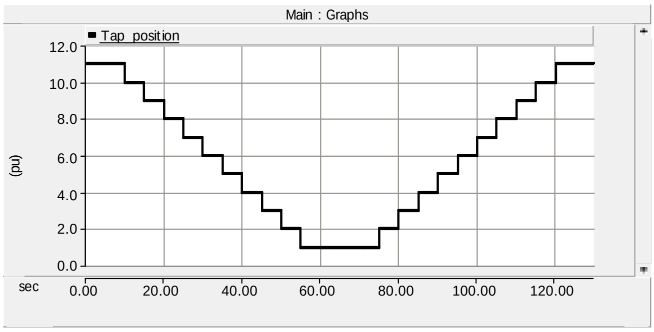

Figure 7: transformer tap position

Above graph shows ULTC transformer tap position. When the voltage was dipped, transformer acts by decreasing tap position. The maximum/minimum tap position that can be increased or decreased from the nominal of ULTC is 10. Therefore, transformer reaches its minimum tap position at 55s. Since bus 9 voltage goes beyond nominal voltage after the load is removed, transformer tap position is increased to reduce the voltage. Transformer tap position is directly related to turns ratio and it is inversely propositional to tap pu. When voltage dip is sensed, to reduce the turns ratio, tap position is reduced. When over voltage is sensed, tap position is increased to increase the turns ratio.

b) Unstable condition

From PV analysis, it is observed that system becomes unstable when load at bus 9 reaches beyond 750 MW. To simulate an unstable condition, an extra load (in addition to existing load) of 900 MW and 30 MVAR was added to bus 9 at 5s until 70s. Effect of under load tap changing transformer action was observed. Obtained results are shown below.

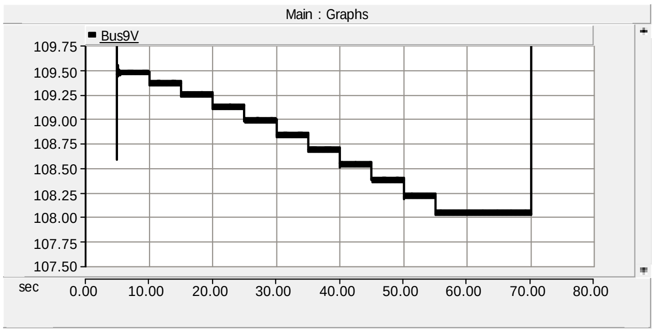

Figure 8: Voltage at bus 9(expanded)

As seen from the above graph, when ULTC tries to restore the voltage at bus 9 after the disturbance, the voltage is further reduced 109.75 kV to 108.25 kV.

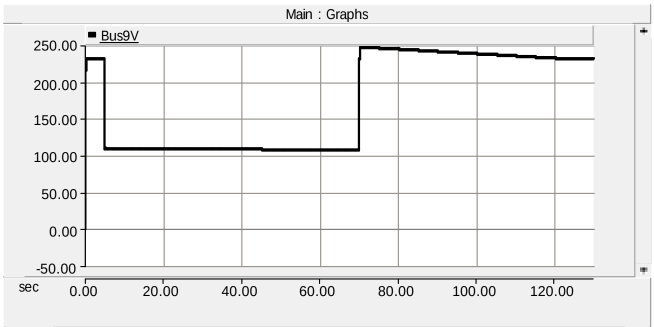

Figure 9: Voltage at bus 9

After extra load was removed, voltage is brought down using ULTC as similar to stable condition.

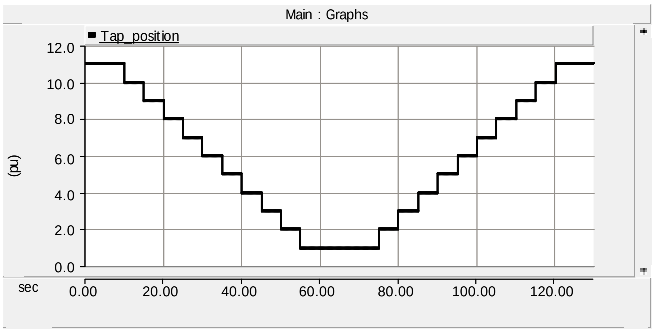

Figure 10: Transformer tap position

It is observed from the above graph that ULTC transformer acted in the same way as the stable condition by reducing tap position (to reduce turns ratio) to increase the voltage at bus 9.

IV. ANALYSIS OF RESULTS

6 load buses in IEEE 9 bus system were ranked from strongest to weakest using 5 techniques. Summary of these results are given below.

Table 7: Summary of results

Method	***Result***
PV	Bus 6 >bus 8 >bus 4> bus 7> bus 5 >bus 9
QV	Bus 6> bus 4> bus 8> bus 7> bus 5> bus 9
VQ sensitivity analysis	Bus 4> bus 8> bus 6> bus 7> bus 9> bus 5
Eigenvalue decomposition	Bus 4> bus 8> bus 6> bus 5> bus 9> bus 7
Singular value decomposition	Bus 4> bus 8> bus 6> bus 5> bus 9 >bus 7
Dynamic Analysis	Bus 8>Bus 4> Bus 6> Bus 7> Bus 9 > Bus 5

From results obtained from section 7 it can be concluded that the action of ULTC under unstable condition aggregates voltage collapse of a bus bar.

V. CONCLUSION

Six different techniques of voltage stability analysis are discussed in this paper. Ranking of buses can slightly differ in IEEE 9 bus system using methods 1-5. Results obtained using Eigenvalue decomposition technique and singular value decomposition technique are identical. Very close values can be observed for diagonal elements of J4 and J_R because in J4 effect of active power in voltage is disregarded. This is a good assumption since when computing $J_{R,}$ active power is assumed to be constant. A dynamic study will give most accurate results for a voltage stability analysis because it accounts accurate modeling of all distribution level devices (ex ULTCs). However, a dynamic study is time consuming in large systems. Time consumed in voltage stability analysis can be reduced by modeling appropriate dynamic equivalents of rest of the power system. By doing this, detailed modeling of area of interest can be preserved while bringing down simulation times. Above methods are impractical for online voltage stability applications as detailed model of the system is required. For such applications measurement based analysis techniques should be developed. Finally, impact of inverter based resources such as wind and solar should be assessed in voltage stability of power systems.

VI. REFERENCES

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